

# Strange Quark Matter Attached to String Cosmology in FRW Space-Time

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Received: 18 April 2011 / Accepted: 28 June 2011 / Published online: 26 July 2011  
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**Abstract** We have obtained FRW cosmological model with strange quark matter attached to the string cloud in general relativity. The model is obtained with the help of special law of variation for Hubble parameter proposed by Bermann (Nuovo Cimento 74B:182, 1983). Also, some physical and kinematics properties of the model are discussed. The results are analogous to results obtained by Yilmaz (Gen. Relativ. Gravit. 38:1397–1406, 2006).

**Keywords** FRW space time · Quark matter · Cosmic strings

## 1 Introduction

In particle physics and cosmology, to know the exact physical situation at the very early stages of the formation of our universe, the string theory is proving to be a useful concept before the creation of the particle in the universe. Because particles can be understood as different states of a single unified field theory in which all particles and fundamental forces of nature to unify gravity are related through a broken symmetry. Certain grand unified field theory predicts topological defects which might have been formed in the phase transition of the universe. The topological stable defects Kibble [3] which occur during the phase transition (as the universe passes through its critical temperature) are identified as strings. The other topological defects are monopoles and domain walls. Spontaneous symmetry breaking is an old idea, described within the particle physics in terms of the Higgs field mechanism. The symmetry is called spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian density. Thus, the vacuum expectation value of the Higgs field is non-zero. In quantum field theories, broken symmetries are restored at high enough temperatures. One of the interesting consequences of the first order phase transition from quark phase to hadron phase in the early universe is the formation of strange quark matter and it has been attracting much interest (Witten [4], Fahri and Jaffe [5]). It is plausible to

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attach strange quark matter to the string cosmology. Because string is free to vibrate and different vibration modes of the string represent the different particle types, since the different modes are observed as different masses or spins. One mode of vibration makes the string appears as an electron and as proton. There is a vibration mode describing the graviton, the particle carrying the force of gravity which play a vital role in string cosmology. In this respect, many authors like Bodmer [6], Alcock et al. [7], Haensel et al. [8], Yilmaz [2, 9] and Yavuz et al. [10] have been studied to the quark matters attached to the topological defects in general relativity.

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Another important phase transition during the phase transitions of the Universe Quark Gluon plasma (QGP) implies hadron gas (called quark-hadron phase transition) when cosmic temperature was  $T \approx 200$  MeV. Itoh [11], Bodmar [6] have formed two ways for creation of strange quark matter. One is the quark-hadron phase transition in the early universe and another is the conversion of neutron stars into strange ones at ultrahigh density. In strong interaction theories it is supposed that breaking of physical vacuum takes place inside hadrons to form quark bag model. As a result vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. The study of strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion has been done by Yavuz et al. [10]. Letelier [12] and Stachel [13] have proposed the study of general relativistic treatment of strings. Letelier [14] has solved Einstein field equations for a cloud of massive strings and obtained cosmological models in Bianchi type-I and Kantowski-Sachs space-time. Strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density  $B_c$  (called as the bag constant). In the framework of this model, the quark matter is composed of mass less  $u$  and  $d$  quarks, massive  $s$  quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are mass less and non interacting. Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3}, \quad (1)$$

where  $\rho_q$  is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c. \quad (2)$$

But the total pressure is

$$p = p_q - B_c, \quad (3)$$

Mak and Harko [15] have studied charged strange quark matter in the spherically symmetric space-time admitting conformal motion. In this paper, we have solved Einstein's field equations for FRW space time with strange quark matter attached to the string cloud.

## 2 The Metric and Field Equations

We consider the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) line element in the form

$$ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (4)$$

where  $R(t)$  is the scale factor,  $K$  is the curvature parameter which can take the values  $K = 0, -1, +1$  for flat, open and closed universe respectively and the signature of the metric is  $(+, -, -, -)$ .

The energy momentum tensor for string cloud (Letelier [14]) is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j. \quad (5)$$

Here  $\rho$  is the rest energy density for the cloud of strings with particles attached to them and  $\rho_s$  is the string tension density. They are related by

$$\rho = \rho_p + \rho_s, \quad (6)$$

where  $\rho_p$  is the particle energy density. We know that string is free to vibrate. The different vibrational modes of the string represent the different types of particles because these different modes are seen as different masses or spins. Therefore, here we will take quarks instead of particles in the string cloud. Hence we consider strange quark matter energy density instead of particle energy density in the string cloud. In this case from (6), we get

$$\rho = \rho_q + \rho_s + B_c. \quad (7)$$

From (5) and (6), (Yavuz et al. [10]) we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_s + B_c) u_i u_j - \rho_s x_i x_j, \quad (8)$$

where  $u_i$  is the four velocity of the particles and  $x_i$  is the unit space like vector representing the direction of string.

We have  $u_i$  and  $x_i$  with satisfying conditions:

$$u_i u^i = -x_i x^i = 1 \quad \text{and} \quad u^i x_i = 0. \quad (9)$$

The Einstein's field equations (with gravitational units  $8\pi G = C = 1$ ) read as

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j, \quad (10)$$

where  $R_i^j$  is the Ricci tensor and  $R = g^{ij} R_{ij}$  is the Ricci scalar.

Using (5) to (10), the field equations (10) for metric (4) can be written as

$$2 \frac{R_{44}}{R} + \frac{R_4^2}{R} + \frac{K}{R^2} = \rho_s, \quad (11)$$

$$3 \frac{R_4^2}{R^2} + \frac{3K}{R^2} = \rho, \quad (12)$$

where the subscript '4' after  $R$  denotes ordinary differentiation with respect to  $t$ .

Equations (11)–(12) are highly non-linear two differential equations in three unknowns  $R$ ,  $\rho_s$  and  $\rho$ .

For the complete determination of these field equations, we use a special law of variation of Hubble parameter, proposed by Bermann [1] as

$$H = DR^{-m}, \quad (13)$$

where  $H$  is the Hubble parameter defined by

$$H = \frac{R_4}{R}, \quad (14)$$

$D$  and  $m (\neq 0)$  are constants.

Using (13) and (14), we obtain

$$R = [m(At + B)]^{\frac{1}{m}}, \quad (15)$$

where  $A$  and  $B$  are taken to be positive constant of integration.

Thus FRW cosmological model by using (15) in General Relativity can be written (after suitable choice of coordinates and constants of integration) as

$$ds^2 = dt^2 - [m(At + B)]^{\frac{2}{m}} \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (16)$$

This model has no initial singularity and represents expanding universe.

Using (11), (12) and (15), we obtain

String tension density

$$\rho_s = \frac{3A^2 - 2m}{[m(At + B)]^2} + \frac{K}{[m(At + B)]^{\frac{2}{m}}}, \quad (17)$$

String energy density

$$\rho = \frac{3A^2}{[m(At + B)]^2} + \frac{3K}{[m(At + B)]^{\frac{2}{m}}}. \quad (18)$$

String particle density  $= \rho_p = \rho - \rho_s$ ,

$$\rho_p = \frac{2m}{[m(At + B)]^2} + \frac{2K}{[m(At + B)]^{\frac{2}{m}}}. \quad (19)$$

Quark energy density

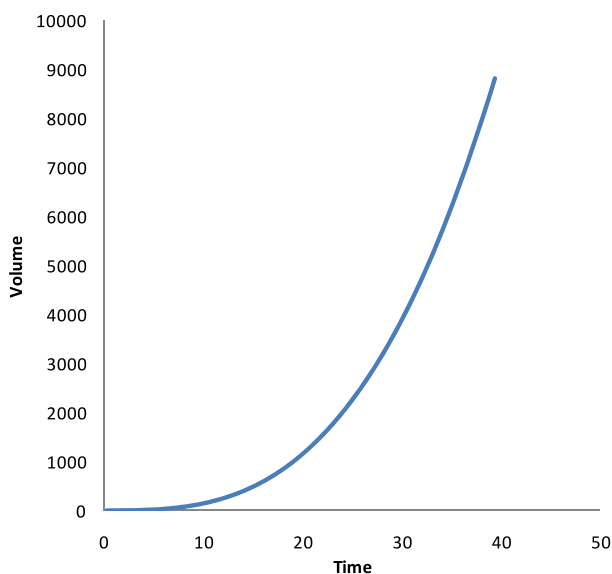
$$\rho_q = \frac{2m}{[m(At + B)]^2} + \frac{2K}{[m(At + B)]^{\frac{2}{m}}} - B_c. \quad (20)$$

Quark pressure

$$p_q = \frac{2m}{[m(At + B)]^2} + \frac{2K}{[m(At + B)]^{\frac{2}{m}}} - \frac{B_c}{3}. \quad (21)$$

The model (16) has no initial singularity, while the tension density and energy density of the string given by (17) and (18) possess initial singularities. However, as  $t$  increases these

**Fig. 1** Spatial volume vs time. We have assume  $A = 1$ ,  $K = 1$ ,  $r = 0.5$ ,  $\theta = 30^\circ$



singularities vanish. At initial epoch ( $t = 0$ ) quark pressure and density are infinite, further both decreases as  $t$  increases. Therefore, we do not have any exact knowledge of the state of the cosmic strings and quark matter at the initial moment of creation of the universe.

### 3 Some Physical Properties

The physical quantities that are important in cosmology are spatial volume  $v^3$ , the expansion scalar  $\theta$ , shear scalar  $\sigma^2$  which have the following expressions for the model (16) as given below:

$$\text{Spatial volume} = \frac{A^3 t^3 r^2 \sin \theta}{(1 - Kr^2)^{\frac{1}{2}}}. \quad (22)$$

It may be observed here that at an initial epoch ( $t = 0$ ), the proper volume will be zero, whereas when  $t \rightarrow \infty$ , the spatial volume becomes infinitely large (Fig. 1).

Scalar expansion

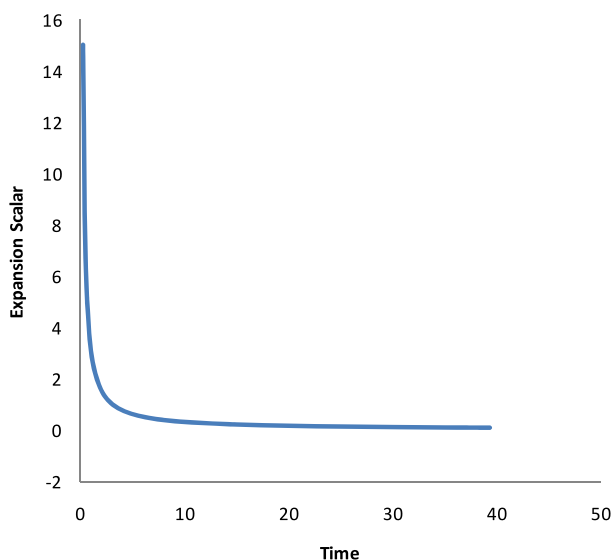
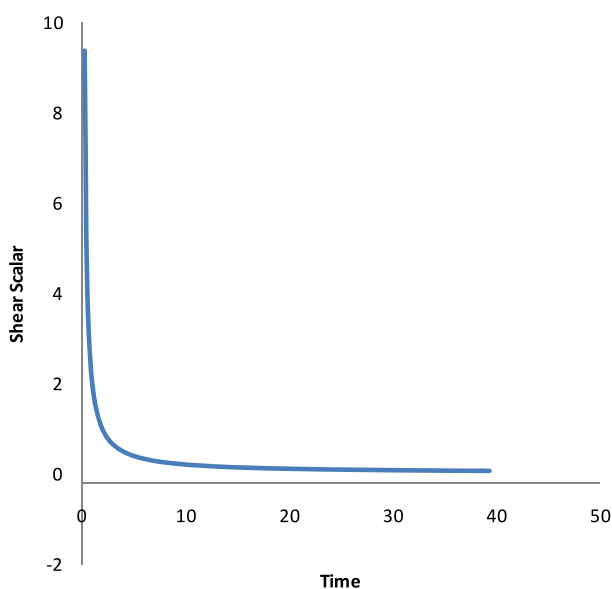
$$\begin{aligned} \theta &= U_{;i}^i, \\ &= \frac{3}{t}. \end{aligned} \quad (23)$$

The expansion scalar  $\theta$  to infinity as  $t \rightarrow 0$ , whereas when  $t \rightarrow \infty$ , the expansion scalar tends to zero (Fig. 2).

Shear scalar

$$\begin{aligned} \sigma^2 &= \frac{1}{2} \sigma^{ij} \sigma_{ij}, \\ &= \frac{7}{2t^2}. \end{aligned} \quad (24)$$

The shear scalar tends to infinity as  $t \rightarrow 0$ , whereas when  $t \rightarrow \infty$ , shear scalar tends to zero (Fig. 3). Also, since  $\lim_{t \rightarrow \infty} (\frac{\sigma^2}{\theta^2}) \neq 0$  being independent of cosmic time implies that the

**Fig. 2** Expansion scalar vs time**Fig. 3** Shear scalar vs time

model does not approach isotropy for large values of  $t$ . The model is expanding shearing, non-rotating and has no initial singularities.

#### 4 Conclusion

It is interesting to note that as  $t$  gradually increases, the scalar expansion  $\theta$  and shear scalar decrease and finally they vanish when  $t \rightarrow \infty$ . This result is consistent with the results obtained by Back et al. [16], Adams et al. [17], Adcox et al. [18] at Brookhaven National

Laboratory and also by Yilmaz [2]. For our model, we get  $\frac{\sigma}{\theta} \approx 0.622$  which is considerably greater than its present value. The present upper limit of  $\frac{\sigma}{\theta}$  is  $10^{-5}$ . This fact implies that our solution represents the early stages of evolution of the universe which is analogous to result obtained by Yilmaz [2].

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